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On Chidume's Open Questions

LEI DENG

Department of Mathematics, Chongqing Teacher's College, Yongchuan 632168, Sichuan, People's Republic of China

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Suppose $X = L_p$ (or l_p), $p \geqslant 2$. Let $T: X \to X$ be a Lipschitzian and strongly accretive map with constant $k \in (0, 1)$ and Lipschitz constant L. Define $S: X \to X$ by $Sx = f - Tx + x$. For arbitrary $x_0 \in X$, the sequence $\{x_n\}_{n=1}^{\infty}$ is defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)\, x_n + \alpha_n S y_n, \\ y_n &= (1-\beta_n)\, x_n + \beta_n S x_n, \qquad n \geqslant 0, \end{aligned}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ are two real sequences satisfying:

(i) $0 \leqslant \alpha_n \leqslant (k + k\beta_n - L^2\beta_n)\{w[1 - 2k\beta_n + \beta_n^2(w + 2k - 1)] + 2k + 2k\beta_n - 2L^2\beta_n - 1\}^{-1}$ for each n,

(ii) $0 \leqslant \beta_n \leqslant k/(2L^2)$ for each n,

(iii) $\sum_n \alpha_n = \infty$.

Then $\{x_n\}_{n=1}^{\infty}$ converges strongly to the unique solution of $Tx = f$. Moreover, if $\alpha_n = (k + k\beta - L^2\beta)\{w[1 - 2k\beta + \beta^2(w + 2k - 1)] + 2k + 2k\beta - 2L^2\beta - 1]^{-1}$ and $\beta_n = \beta$ for each n and some $\beta \in [0, k/(2L^2)]$, then

$$\|x_{n+1} - q\| \leqslant \rho^{n/2}\, \|x_1 - q\|,$$

where q denotes the solution of $Tx = f$ and

$$\begin{aligned} \rho = 1 - (k + k\beta - L^2\beta)^2 \, \{&w[1 - 2k\beta + \beta^2(w + 2k - 1)] \\ &+ 2k + 2k\beta - 2L^2\beta - 1\}^{-1} \in (0, 1). \end{aligned}$$

A related result deals with the iterative approximation of Lipschitz strongly pseudocontractive maps in X.

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1. Introduction and Preliminaries

Let X be a real normed linear space. A mapping T with domain $D(T)$ and range $R(T)$ in X is said to be accretive [2] if the inequality

$$\|x-y\| \leqslant \|x-y+t(Tx-Ty)\| \tag{1}$$

holds for each x and y in $D(T)$ and for all $t \geqslant 0$. If (1) holds only for some $t>0$, T is said to be monotone [14]. If X is a Hilbert space, the accretive condition (1) reduces to

$$\operatorname{Re} \langle Tx-Ty, x-y\rangle \geqslant 0 \tag{2}$$

for all x, y in X. The accretive operators were introduced independently by F. E. Browder [2] and T. Kato [14] in 1967. An ealy fundamental result in the theory of accretive operators, due to Browder, states that the initial value problem

$$du/dt + Tu = 0, \qquad u(0) = u_0 \tag{3}$$

is solvable if T is locally Lipschitzian and accretive on X.

For a Banach space X we denote by J the normalized duality map from X to 2^{X^*} given by

$$Jx = \{f^* \in X^*: \|f^*\|^2 = \langle x, f^*\rangle\},$$

where X^* denotes the dual space of X and $\langle\ ,\ \rangle$ denotes the generalized duality pairing. It is well known that if X^* is strictly convex, then J is single-valued, and if X^* is uniformly convex, then J is uniformly continuous on bounded sets (see, e.g., [3]).

Let K be a nonempty subset of a Banach space X. A mapping $T: K \to X$ is said to be strongly accretive if for each x, y in K there exists $w \in J(x-y)$ such that

$$\langle Tx-Ty, w\rangle \geqslant k\,\|x-y\|^2 \tag{4}$$

for some real constant $k>0$. Without loss of generality we assume that $k \in (0, 1)$. Strongly accretive mappings are sometimes also called strictly accretive. These mappings have been studied by several authors (e.g., [1, 3, 7, 12, 16, 18, 19]).

Let K be a nonempty subset of a Banach space X. A mapping $T: K \to X$ is said to be strictly pseudocontractive if there exists $t>1$ such that the inequality,

$$\|x-y\| \leqslant \|(1+r)(x-y) - rt(Tx-Ty)\| \tag{5}$$

holds for all x, y in K and $r > 0$. If, in the above definition, $t = 1$, then T is said to be a pseudocontractive mapping. Strictly pseudocontractive mappings have been studied by various authors (see, e.g., [1, 6, 7]).

The Ishikawa iteration process (see, e.g., [7, 13, 23]) is defined as follows: For K a convex subset of a Banach space X, and T a mapping of K into itself, the sequence $\{x_n\}_{n=1}^{\infty}$ in K is defined by

$$\begin{aligned} x_0 \in K, \qquad x_{n+1} &= (1-\alpha_n)\, x_n + \alpha_n T y_n, \\ y_n &= (1-\beta_n)\, x_n + \beta_n T x_n, \qquad n \geqslant 0, \end{aligned} \tag{6}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ satisfy $0 \leqslant \alpha_n, \beta_n \leqslant 1$ for all n, and $\sum_n \alpha_n = \infty$.

The Mann iteration process (see, e.g., [7, 15, 23]) is similar to the Ishikawa iteration process above but with $\beta_n \equiv 0$.

The iteration processes described above have been studied extensively by several authors for approximating the fixed points of several nonlinear mappings and for approximating solutions of several nonlinear operator equations in Banach space (see, e.g., [4, 5, 7–12, 15–24]). For a comparison of the two iterative schemes the reader may consult [23].

For the remainder of this paper, X denotes an L_p *(or* l_p*)* space with $p \geqslant 2$, and the single-valued duality map is denoted by j. The Lipschitz constant of T is denoted by L $(\geqslant 1)$ and w denotes $(p-1)\, L^2$, and the constant appearing in the definition of a strongly accretive map is denoted by $k \in (0, 1)$. In [6, 7] the following inequality is proved:

$$\|x+y\|^2 \leqslant (p-1)\, \|x\|^2 + \|y\|^2 + 2\langle x, j(y)\rangle \tag{7}$$

for all x, y in X.

In [7], using the above inequality, C. E. Chidume proved (Theorem 1) that the Mann iteration process converges strongly to a solution of the equation $Tx = f$ when T is Lipschitzian and strongly accretive. A related result (Theorem 2) deals with the iterative approximation of the fixed point of the class of Lipschitz strictly pseudocontractive mappings. At the same time, he put forth the following question.

Can the Ishikawa iteration process be extended to Theorems 1 and 2?

In this paper, we answer positively the above question in the more general setting, removing the restriction $\alpha_n \leqslant \beta_n$ and $\lim_n \beta_n = 0$.

2. Main Results

THEOREM 1. *Let $T\colon X \to X$ be a Lipschitzian and strongly accretive map. Define $S\colon X \to X$ by $Sx = f - Tx + x$. For arbitrary $x_0 \in X$, the sequence $\{x_n\}_{n=1}^{\infty}$ is defined by*

$$x_{n+1} = (1-\alpha_n)\, x_n + \alpha_n S y_n,$$
$$y_n = (1-\beta_n)\, x_n + \beta_n S x_n, \qquad n \geqslant 0, \tag{8}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ *are two real sequences satisfying*:

(i) $0 \leqslant \alpha_n \leqslant (k + k\beta_n - L^2\beta_n)\{w[1 - 2k\beta_n + \beta_n^2(w + 2k - 1)] + 2k + 2k\beta_n - 2L^2\beta_n - 1)\}^{-1}$ *for each n*,

(ii) $0 \leqslant \beta_n \leqslant k/(2L^2)$ *for each n*,

(iii) $\sum_n \alpha_n = \infty$.

Then $\{x_n\}_{n=1}^{\infty}$ *converges strongly to the unique solution of* $Tx = f$. *Moreover, if* $\alpha_n = (k + k\beta - L^2\beta)\{w[1 - 2k\beta + \beta^2(w + 2k - 1)] + 2k + 2k\beta - 2L^2\beta - 1\}^{-1}$ *and* $\beta_n = \beta$ *for each n and some* $\beta \in [0, k/(2L^2)]$, *then*

$$\|x_{n+1} - q\| \leqslant \rho^{n/2}\, \|x_1 - q\|,$$

where q denotes the solution of $Tx = f$ *and*

$$\rho = 1 - (k + k\beta - L^2\beta)^2\, \{w[1 - 2k\beta + \beta^2(w + 2k - 1)] + 2k + 2k\beta - 2L^2\beta - 1\}^{-1} \in (0, 1).$$

Proof. The existence of a solution to $Tx = f$ follows from Morales [16]. Let q be a solution. Observe that S is Lipschitzian with the same Lipschitz constant L, and q is a fixed point of S. It follows from (4) that

$$\langle Sx_n - Sq, j(x_n - q)\rangle = -\langle Tx_n - Tq, j(x_n - q)\rangle + \langle x_n - q, j(x_n - q)\rangle \leqslant (1-k)\, \|x_n - q\|^2. \tag{9}$$

Using (7) and (9), we have

$$\begin{aligned}
\|y_n - q\|^2 &= \|\beta_n(Sx_n - Sq) + (1-\beta_n)(x_n - q)\|^2 \\
&\leqslant (p-1)\, \beta_n^2\, \|Sx_n - Sq\|^2 + (1-\beta_n)^2\, \|x_n - q\|^2 \\
&\quad + 2\beta_n(1-\beta_n)\langle Sx_n - Sq, j(x_n - q)\rangle \\
&\leqslant [(p-1)\, L^2\beta_n^2 + (1-\beta_n)^2 + 2\beta_n(1-\beta_n)(1-k)]\, \|x_n - q\|^2 \\
&\leqslant [1 - k\beta_n + \beta_n^2(w + 2k - 1)]\, \|x_n - q\|^2.
\end{aligned} \tag{10}$$

By (4) and $Tq = f$, we have

$$\begin{aligned}
&\langle y_n - q, j(x_n - q)\rangle \\
&= \langle -\beta_n Tx_n + x_n + \beta_n f - q, j(x_n - q)\rangle \\
&= -\beta_n\langle Tx_n - Tq, j(x_n - q)\rangle + \langle x_n - q, j(x_n - q)\rangle \\
&\leqslant -k\beta_n\, \|x_n - q\|^2 + \|x_n - q\|^2 \\
&= (1 - k\beta_n)\, \|x_n - q\|^2
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\langle Sy_n - Sq, j(x_n - q)\rangle \\
&\quad = \langle -Ty_n + Tq + y_n - q, j(x_n - q)\rangle \\
&\quad = \langle Tx_n - Ty_n, j(x_n - q)\rangle - \langle Tx_n - Tq, j(x_n - q)\rangle \\
&\qquad + \langle y_n - q, j(x_n - q)\rangle \\
&\quad \leqslant L\,\|x_n - y_n\|\,\|x_n - q\| - k\,\|x_n - q\|^2 + (1 - k\beta_n)\,\|x_n - q\|^2 \\
&\quad \leqslant L\beta_n\,\|Tx_n - Tq\|\,\|x_n - q\| + (1 - k - k\beta_n)\,\|x_n - q\|^2 \\
&\quad \leqslant (1 - k - k\beta_n + L^2\beta_n)\,\|x_n - q\|^2. \qquad (12)
\end{aligned}$$

It follows from (7), (8), (10), and (12) that

$$\begin{aligned}
&\|x_{n+1} - q\|^2 \\
&\quad = \|\alpha_n(Sy_n - Sq) + (1 - \alpha_n)(x_n - q)\|^2 \\
&\quad \leqslant (p-1)\alpha_n^2\,\|Sy_n - Sq\|^2 + (1 - \alpha_n)^2\,\|x_n - q\|^2 \\
&\qquad + 2\alpha_n(1 - \alpha_n)\langle Sy_n - Sq, j(x_n - q)\rangle \\
&\quad \leqslant (p-1)\,L^2\alpha_n^2\,\|y_n - q\|^2 + (1 - \alpha_n)^2\,\|x_n - q\|^2 \\
&\qquad + 2\alpha_n(1 - \alpha_n)(1 - k - k\beta_n + L^2\beta_n)\,\|x_n - q\|^2 \\
&\quad \leqslant w\alpha_n^2[1 - 2k\beta_n + \beta_n^2(w + 2k - 1)]\,\|x_n - q\|^2 + [(1 - \alpha_n)^2 \\
&\qquad + 2\alpha_n(1 - \alpha_n)(1 - k - k\beta_n + L^2\beta_n)]\,\|x_n - q\|^2 \\
&\quad = \{1 - 2(k + k\beta_n - L^2\beta_n)\,\alpha_n + \{w[1 - 2k\beta_n + \beta_n^2(w + 2k - 1)] \\
&\qquad + 2k + 2k\beta_n - 2L^2\beta_n - 1\}\,\alpha_n^2\}\,\|x_n - q\|^2. \qquad (13)
\end{aligned}$$

By condition (ii), i.e., $0 \leqslant \beta_n \leqslant k/(2L^2)$, we have

$$\begin{aligned}
&w[1 - 2k\beta_n + \beta_n^2(w + 2k - 1)] + 2k + 2k\beta_n - 2L^2\beta_n - 1 \\
&\quad \geqslant w(1 - 2k\beta_n) + 2k + 2k\beta_n - 2L^2\beta_n - 1 \\
&\quad \geqslant (w - 1)(1 - 2k\beta_n) + 2k - 2L^2\beta_n \\
&\quad \geqslant k > 0,
\end{aligned}$$

and $k + k\beta_n - L^2\beta_n \geqslant k + (k - L^2)k/(2L^2) > 0$.

So using condition (i), we obtain

$$\begin{aligned}
\|x_{n+1} - q\|^2 &\leqslant [1 - 2(k + k\beta_n - L^2\beta_n)\,\alpha_n + (k + k\beta_n - L^2\beta_n)\,\alpha_n]\,\|x_n - q\|^2 \\
&\leqslant [1 - (k + k\beta_n - L^2\beta_n)\,\alpha_n]\,\|x_n - q\|^2. \qquad (14)
\end{aligned}$$

Thus, again using condition (ii), we have

$$\begin{aligned}\|x_{n+1}-q\|^2 &\leqslant \{1-[k+(k-L^2)k/(2L^2)]\,\alpha_n\}\,\|x_n-q\|^2\\ &\leqslant (1-2^{-1}k\alpha_n)\,\|x_n-q\|^2\\ &\leqslant \exp(-2^{-1}k\alpha_n)\,\|x_n-q\|^2. \end{aligned} \tag{15}$$

Iterating the last inequality from $n=1$ to N we obtain

$$\|x_{N+1}-q\|^2 \leqslant \exp\left(-2^{-1}k\sum_1^N \alpha_n\right)\|x_1-q\|^2 \to 0 \qquad \text{as} \quad N\to\infty$$

since $\sum_n \alpha_n=\infty$. Hence $\{x_n\}_{n=1}^{\infty}$ converges strongly to q.
For uniqueness, see [7].

Error Estimate. If $\alpha_n=(k+k\beta-L^2\beta)\{w[1-2k\beta+\beta^2(w+2k-1)]+2k+2k\beta-2L^2\beta-1\}^{-1}$ and $\beta_n=\beta$ for each n and some $\beta\in[0,k/(2L^2)]$, then from inequality (14),

$$\begin{aligned}\|x_{n+1}-q\|^2 &\leqslant (1-(k+k\beta-L^2\beta)^2\,\{w[1-2k\beta+\beta^2(w+2k-1)]\\ &\qquad +2k+2k\beta-2L^2\beta-1\}^{-1})\,\|x_n-q\|^2\\ &\leqslant (1-(k+k\beta-L^2\beta)^2\,\{w[1-2k\beta+\beta^2(w+2k-1)]\\ &\qquad +2k+2k\beta-2L^2\beta-1\}^{-1})^n\,\|x_1-q\|^2,\end{aligned}$$

so that

$$\|x_{n+1}-q\|\leqslant \rho^{n/2}\,\|x_1-q\|,$$

where ρ is as defined, completing the proof of the theorem.

Remark 1. Theorem 1 of [7] is the special case of Theorem 1 for $\beta_n\equiv 0$.

THEOREM 2. *Suppose K is a nonempty closed bounded and convex subset of X and $T\colon K\to K$ is a Lipschitz strictly pseudocontractive mapping of K into itself. Let the sequence $\{x_n\}_{n=1}^{\infty}$ be defined by*

$$\begin{aligned} x_0\in K,\qquad x_{n+1}&=(1-\alpha_n)\,x_n+\alpha_n Ty_n,\\ y_n&=(1-\beta_n)\,x_n+\beta_n Tx_n,\qquad n\geqslant 0,\end{aligned}\tag{16}$$

where $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ are two real sequences satisfying:

(i) $0 \leqslant \alpha_n \leqslant (s - L\beta_n - L^2\beta_n)\{w[1 - 2s\beta_n + \beta_n^2(w + 2s - 1)] + 2s - 2L\beta_n - 2L^2\beta_n - 1)\}^{-1}$ *for each* n,

(ii) $0 \leqslant \beta_n \leqslant s/(2L^2 + 4s)$ *for each* n,

(iii) $\sum_n \alpha_n = \infty$,

where $s = (t-1)/t$ *and* $t > 1$ *is the constant appearing in inequality* (5). *Then* $\{x_n\}_{n=1}^{\infty}$ *converges strongly to the unique fixed point of* T.

Proof. The existence of a fixed point follows from Deimling [8]. Let q denote a fixed point of T. Since T is strictly pseudocontractive then $(I - T)$ is strongly accretive (see, e.g., [1] or [6]). So, for each x, y in K,

$$\operatorname{Re}\langle (I-T)x - (I-T)y, j(x-y)\rangle \geqslant s\,\|x-y\|^2,$$

where $s = (t-1)/t$. Hence

$$\begin{aligned}\langle Tx_n - Tq, j(x_n - q)\rangle &= -\langle (I-T)\,x_n - (I-T)q, j(x_n - q)\rangle \\ &\quad + \langle x_n - q, j(x_n - q)\rangle \\ &\leqslant (1-s)\,\|x_n - q\|^2. \qquad (17)\end{aligned}$$

Thus, using (7) and (17), we have

$$\begin{aligned}&\|y_n - q\|^2 \\ &\quad = \|\beta_n(Tx_n - Tq) + (1-\beta_n)(x_n - q)\|^2 \\ &\quad \leqslant [w\beta_n^2 + (1-\beta_n)^2]\,\|x_n - q\|^2 + 2\beta_n(1-\beta_n)\langle Tx_n - Tq, j(x_n - q)\rangle \\ &\quad \leqslant [1 - 2s\beta_n + \beta_n^2(w + 2s - 1)]\,\|x_n - q\|^2 \qquad (18)\\ &\langle y_n - q, j(x_n - q)\rangle \\ &\quad = \langle \beta_n(Tx_n - Tq) + (1-\beta_n)(x_n - q), j(x_n - q)\rangle \\ &\quad = \beta_n\langle Tx_n - Tq, j(x_n - q)\rangle + (1-\beta_n)\langle x_n - q, j(x_n - q)\rangle \\ &\quad \leqslant (1 - s\beta_n)\,\|x_n - q\|^2 \qquad (19)\\ &\langle Ty_n - Tq, j(x_n - q)\rangle \\ &\quad = \langle Ty_n - Tx_n, j(x_n - q)\rangle + \langle Tx_n - Tq, j(x_n - q)\rangle \\ &\quad \leqslant \|Ty_n - Tx_n\|\,\|x_n - q\| + (1-s)\,\|x_n - q\|^2 \\ &\quad \leqslant L\,\|y_n - x_n\|\,\|x_n - q\| + (1-s)\,\|x_n - q\|^2 \\ &\quad \leqslant L\beta_n\,\|(Tx_n - Tq) + (q - x_n)\|\,\|x_n - q\| + (1-s)\,\|x_n - q\|^2 \\ &\quad \leqslant (1 - s + L\beta_n + L^2\beta_n)\,\|x_n - q\|^2. \qquad (20)\end{aligned}$$

From (16), using (7), (18), and (20),

$$\begin{aligned}
&\|x_{n+1} - q\|^2 \\
&\quad = \|\alpha_n(Ty_n - Tq) + (1-\alpha_n)(x_n - q)\|^2 \\
&\quad \leqslant w\alpha_n^2 \|y_n - q\|^2 + (1-\alpha_n)^2 \|x_n - q\|^2 \\
&\qquad + 2\alpha_n(1-\alpha_n)\langle Ty_n - Tq, j(x_n - q)\rangle \\
&\quad \leqslant \{1 - 2(s - L\beta_n - L^2\beta_n)\,\alpha_n + \{w[1 - 2s\beta_n + \beta_n^2(w + 2s - 1)] \\
&\qquad + 2s - 2L\beta_n - 2L^2\beta_n - 1\}\,\alpha_n^2\}\,\|x_n - q\|^2. \qquad (21)
\end{aligned}$$

The remainder of the proof is similar to that of Theorem 1.

Remark 2. Theorem 2 of [7] is the special case of Theorem 2 for $\beta_n \equiv 0$.

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